Refining the Neutron Star Mass Determination in Six Eclipsing X-ray Pulsar Binaries

Meredith L. Rawls
Jerome A. Orosz
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Overview

- X-ray pulsar and neutron star primer
- Introduction to the six systems
- How masses have been determined in the past (analytic method)
- Our new and improved numerical method using the Eclipsing Light Curve code (ELC)
  - Why this technique is superior to the analytic method
  - How ELC works with MCMC or genetic optimizers
- Incorporating optical light curves
- Results: new values for the neutron star masses
What is an X-ray pulsar?

• “Normal” companion star and neutron star orbiting each other
• X-rays are produced as matter is pulled away from the companion star toward the neutron star
Why study X-ray pulsars?

- Neutron stars are extremely dense collections of matter
- Neutron stars in binaries are easy to detect and study
- An empirical mass range would enable theorists to better understand NS formation and constrain possible equations of state (EoS)
  - A “stiff” EoS puts upper mass limit $\sim 3 \, M_\odot$
  - A “soft” EoS puts upper mass limit $\sim 1.5 \, M_\odot$
  - Formation theory constrains lower mass limit
- Goal of this study: determine the mass of the neutron star in six systems
Meet the six systems

• Vela X-1
  – Eccentric orbit \( e = 0.09 \), \( P = 8.96 \) days
  – Pulsar rotates every 283 seconds
  – Companion star is a B0.5 supergiant

• 4U 1538-52
  – Eccentric orbit \( e \sim 0.18 \), \( P = 3.73 \) days
  – Pulsar rotates every 529 Seconds
  – Companion star is a B0 supergiant

• SMC X-1
  – Circular orbit, \( P = 3.89 \) days
  – Pulsar rotates every 0.71 seconds
  – Companion star is a B0 supergiant
  – Superorbital X-ray cycle observed
Meet the six systems

- **LMC X-4**
  - Circular orbit, $P = 1.41$ days
  - Pulsar rotates every 13.5 seconds, companion O7 III-V star
  - Superorbital X-ray cycle observed

- **Cen X-3**
  - Circular orbit, $P = 2.09$ days
  - Pulsar rotates every 4.84 seconds, companion O6.5 giant

- **Her X-1**
  - Circular orbit, $P = 1.70$ days
  - Pulsar rotates every 1.24 seconds
  - Lower mass companion star ($\sim 2 \, M_\odot$) with variable spectral type
  - Extreme X-ray heating, superorbital cycle observed
Mass determination: analytic approach

- Component masses in terms of “mass functions”

\[
M_{\text{opt}} = \frac{K_x^3 P (1-e^2)^{3/2}}{2 \pi G \sin^3 i} (1+q)^2
\]

\[
M_x = \frac{K_{\text{opt}}^3 P (1-e^2)^{3/2}}{2 \pi G \sin^3 i} \left(1+\frac{1}{q}\right)^2
\]

- Mass ratio is

\[
q = \frac{M_x}{M_{\text{opt}}} = \frac{K_{\text{opt}}}{K_x}
\]
Mass determination: analytic approach

- Measure $K_X$ and $P$ from X-ray pulse timing
  - $a_x \sin i$ is cited in publications:
    \[
    K_X = \frac{2\pi c (1-e^2)^{3/2} a_x \sin i}{P}
    \]
- Measure $K_{opt}$ from optical spectra
Mass determination: analytic approach

• For a spherical companion star, we can relate the eclipse duration, radius, system inclination, and orbital separation

\[
\sin i = \frac{\sqrt{1 - (R/a)^2}}{\cos \theta_e}
\]

– \( \theta_e \) is an angle that represents \textit{half} of the eclipse duration

• But the companion star is NOT spherical!
Mass determination: analytic approach

• Rewrite the radius as a fraction of the “effective Roche lobe radius”

\[
\sin i = \frac{\sqrt{1 - \beta^2(R_L/a)^2}}{\cos \theta_e}
\]

• Use an approximation for \( RL/a \)

\[
\frac{R_L}{a} \approx A + B \log q + C \log^2 q
\]

– Constants \( A, B, \) and \( C \) depend on the ratio of the rotational frequency of the optical companion to the orbital frequency of the system, \( \Omega \)

\[
R = \beta R_L
\]

Roche lobe filling factor
(if the orbit is eccentric, \( \beta \) is defined at periastron)
Mass determination: analytic approach

• Given values of \( P \), \( a_x \sin i \), \( \theta_e \), \( K_{opt} \), \( \Omega \), \( \beta \) (plus \( e \), \( \omega \) if the orbit is eccentric) we can determine the neutron star mass!

  – Can estimate \( \Omega \) from the projected rotational velocity of the companion star, \( v_{rot} \sin i \)
  – Must assume some value for \( \beta \)
  – Expect \( 0.9 < \beta < 1 \)

• Can use a Monte Carlo technique to derive the **most likely mass** (measured input quantities are not known *exactly*)
Examining the approximations, 1

- Computing $R_L/a$
  - Shape and size of Roche lobes depend *only* on the mass ratio $q$ and the parameter $\Omega$

$$\frac{R_L}{a} \approx A + B \log q + C \log^2 q$$

- Compare to result from Eclipsing Light Curve (ELC) code
  - Defines equipotential surfaces based on the gravitational potential at L1
Examining the approximations, 2

- Computing the X-ray eclipse duration, \(2\theta_e\)
  - Depends on the computation of \(R_L/a\)

\[
\sin i = \frac{\sqrt{1 - \beta^2 (R_L/a)^2}}{\cos \theta_e}
\]

- Compare to result from Eclipsing Light Curve (ELC) code
  - Uses the Roche lobe shape of the star rather than a spherical approximation
Examining the approximations, 2

\[ \beta = 1.0 \]

\[ \beta = 0.9 \]
• Difference in eclipse duration can be extreme ($\pm 10^\circ$)
• Directly impacts neutron star mass calculation
Mass determination: numerical method

- Parameter space to search
  - Orbital period, $P$
  - Orbital separation, $a$
  - Mass ratio, $q$
  - Roche lobe filling factor, $\beta$
  - Synchronous rotation parameter, $\Omega$
  - System inclination, $i$
  - Eccentric orbit parameters: $e$, $\omega$

- Fix $P$ and $a_x \sin i$ (known to high accuracy)

$$K_X = \frac{2\pi c \left(1-e^2\right)^{3/2} a_x \sin i}{P}$$

$$q = \left(\frac{K_{opt}}{K_X}\right)$$

$$a = \left(1+\frac{1}{q}\right) c a_x$$

From $v_{rot} \sin i$
Mass determination: numerical method

• We have a six-dimensional parameter space:
  – \((K_{opt}, \beta, \Omega, i, e, \omega)\)

• Can use ELC to form a model binary system when these six parameters are specified
  – Values for each parameter are available from previously published works for all six systems

• Need a way to choose the BEST model…
Mass determination: numerical method

- ELC forms a random set of parameters
- “Fitness” of a model is defined by $\chi^2$
  (lower = better)

$$\chi^2 = \left(\frac{\theta_e(\text{mod}) - \theta_e(\text{obs})}{\sigma_{\theta_e}}\right)^2 + \left(\frac{v_{\text{rot}} \sin i(\text{mod}) - v_{\text{rot}} \sin i(\text{obs})}{\sigma_{v_{\text{rot}} \sin i}}\right)^2 + \left(\frac{K_{\text{opt}}(\text{mod}) - K_{\text{opt}}(\text{obs})}{\sigma_{K_{\text{opt}}}}\right)^2 + \left(\frac{e(\text{mod}) - e(\text{obs})}{\sigma_e}\right)^2 + \left(\frac{\omega(\text{mod}) - \omega(\text{obs})}{\sigma_\omega}\right)^2.$$

(mod) = computed from model
(obs) = observed quantity
$\sigma(\cdot) = 1\sigma$ uncertainty

- One of two “optimizers” is used to construct new parameter sets
- Process is repeated until $\chi^2$ is minimized
Mass determination: numerical method

• ELC can use two different optimizers: Monte Carlo Markov Chain or a genetic algorithm

• Monte Carlo Markov Chain
  – Optimizer takes a “random walk” step for each parameter
  – Given the present state, past and future states are independent
  – Model with highest fitness is the next starting point

• Genetic algorithm
  – Probability of previous models “breeding” is based on fitness
  – Random variations ("mutations") are introduced
  – Models with highest fitness are allowed to “breed”
• One parameter we haven’t constrained: the Roche lobe filling factor, $\beta$

• Can compute models for a range of $\beta$
  – Recall: we expect roughly $0.9 < \beta < 1$
  – System inclination $i$ is inversely correlated with $\beta$

• Preliminary analysis comparing neutron star masses computed **numerically** vs. **analytically**…
Mass determination: numerical method

- Neutron star mass is highly dependent on the choice of $\beta$
- Numerical and analytic results can differ in opposite senses to varying degrees

Need a way to constrain $\beta$...

Optical light curves
Optical light curves

• Ellipsoidal variations
  – Light from companion star changes with orientation

• Light curve shape depends on: \( q, i, \Omega, \beta \)

Already well determined from X-ray eclipse width and \( K \)-velocities

May be constrained with optical light curves!
Optical light curves

- Numerical technique with ELC can be expanded to incorporate new observations

- Modified “fitness” function:

  \[ \chi^2_{\text{new}} = \chi^2 + \sum_{i=1}^{N} \left( \frac{(y_{i, \text{mod}} - y_{i, \text{obs}})}{\sigma_i} \right)^2 \]

  - Set of \( N \) observations with observable quantities \( y_i \)

- Similar terms may be added for additional sets of observations (e.g., radial velocity curve)
Optical light curves

- From previous literature for four systems

- Vela X-1
- LMC X-4
- SMC X-1
- Cen X-3
Optical light curves

- Systems from previous figure
  - Vela X-1, SMC X-1, LMC X-4, Cen X-3
  - All models include an accretion disk for the best fit

- 4U 1538-52
  - We obtained new observations
  - $BVI$ images $\rightarrow$ light curve
  - High resolution spectra $\rightarrow$ radial velocity curve

- Her X-1
  - No optical light curves used due to large uncertainty in $K_{opt}$
  - Previous literature suggests $\beta \approx 1$
Sample final model: Cen X-3
New observations: 4U 1538-52

- Eccentricity $e$ given as 0.08, 0.18 (sometimes $e = 0$ is adopted)

- Argument of periastron $\omega$ given as 244°, 220°

- Obtained $BVI$ images at CTIO
  - 1.3 m SMARTS telescope with the ANDICAM
  - 39 images, June – September 2009

- Obtained high resolution spectra at LCO
  - 6.5 m Clay Magellan telescope with the MIKE spectrograph
  - 21 images, July – August 2009
New Observations: 4U 1538-52

Optical light curve

Radial velocity curve

Velocities calculated via cross-correlation of the spectrum with a model B0 star
### Final results

<table>
<thead>
<tr>
<th></th>
<th>Analytic</th>
<th></th>
<th></th>
<th>Numerical</th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$M_X$ ($M_\odot$)</td>
<td>$i$ (deg)</td>
<td></td>
<td>$M_X$ ($M_\odot$)</td>
<td>$i$ (deg)</td>
<td>$M_{opt}$ ($M_\odot$)</td>
<td>$R_{opt}$ ($R_\odot$)</td>
<td>$f$</td>
</tr>
<tr>
<td>Vela X-1</td>
<td>1.788 ± 0.157</td>
<td>83.6 ± 3.1</td>
<td></td>
<td>1.770 ± 0.083</td>
<td>78.8 ± 1.2</td>
<td>24.00 ± 0.37</td>
<td>31.82 ± 0.28</td>
<td>0.99 ± 0.01</td>
</tr>
<tr>
<td>4U 1538-52</td>
<td>0.875 ± 0.161</td>
<td>83.1 ± 3.5</td>
<td></td>
<td>0.800 ± 0.279</td>
<td>71.5 ± 7.6</td>
<td>19.07 ± 2.71</td>
<td>14.65 ± 3.62</td>
<td>0.84 ± 0.12</td>
</tr>
<tr>
<td>SMC X-1</td>
<td>1.064 ± 0.105</td>
<td>67.8 ± 4.2</td>
<td></td>
<td>1.037 ± 0.085</td>
<td>68.5 ± 5.2</td>
<td>15.35 ± 1.53</td>
<td>15.70 ± 1.36</td>
<td>0.86 ± 0.07</td>
</tr>
<tr>
<td>LMC X-4</td>
<td>1.249 ± 0.094</td>
<td>68.8 ± 3.3</td>
<td></td>
<td>1.285 ± 0.051</td>
<td>67.0 ± 1.9</td>
<td>14.96 ± 0.58</td>
<td>7.76 ± 0.32</td>
<td>0.86 ± 0.03</td>
</tr>
<tr>
<td>Cen X-3</td>
<td>1.473 ± 0.143</td>
<td>67.5 ± 3.2</td>
<td></td>
<td>1.486 ± 0.082</td>
<td>66.7 ± 2.4</td>
<td>22.06 ± 1.37</td>
<td>12.56 ± 0.56</td>
<td>&gt; 0.96</td>
</tr>
<tr>
<td>Her X-1</td>
<td>1.036 ± 0.311</td>
<td>80.5 ± 3.8</td>
<td></td>
<td>1.073 ± 0.358</td>
<td>&gt; 85.9</td>
<td>2.03 ± 0.37</td>
<td>3.76 ± 0.54</td>
<td>–</td>
</tr>
</tbody>
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**Note:**

- For consistency, these analytic values are derived using the $\beta$ values returned by the numerical model rather than a distribution of $0.9 \leq \beta \leq 1$ as in Table 3 and Figure 1.

- The sphere-equivalent radius of the companion star (e.g. see Figure 5).

- The ELC fill factor, defined as the distance from the companion star’s center of mass to the point of the star closest to L1. A value of $f$ maps directly to a value for $\beta$, which is the Roche lobe filling factor expressed in terms of the sphere-equivalent volume radius. For the eccentric systems, both $f$ and $\beta$ are defined at periastron.
Final results

Neutron Star Mass ($M_\odot$)
Final results

- Vela X-1 has a relatively high mass
  - $1.77 \pm 0.08 \, M_\odot$
  - Rules out “soft” equations of state (upper limit $\sim 1.5 \, M_\odot$)
  - Other studies cite a larger value for $K_{opt}$, which gives an even higher mass

- 4U 1538-52 and SMC X-1 have very low masses
  - $0.80 \pm 0.28 \, M_\odot$; $1.04 \pm 0.09 \, M_\odot$ respectively
  - Both are within $1\sigma$ of $1M_\odot$
  - Challenges formation theories of neutron stars in supernovae (expect lower limit $\sim$ Chandrasekhar mass)
Future work

• Improved radial velocity measurements
  – Neutron star mass is proportional to $K_{opt}^3$
  – Difficult to improve for Vela X-1, Her X-1

• Improved eclipse width measurements
  – More time coverage on X-ray observations
  – Need to account for differences in low/high states

[Graph showing X-ray flux for SMC X-1 in high and low states, with data from Coe et al. 2010]
Summary

• Determined neutron star masses for six eclipsing X-ray binary pulsars

• Improved upon previous analytic approach with a new, more accurate numerical technique via ELC

• Incorporated optical light curves into the analysis to constrain the companion star’s Roche lobe filling factor
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• A full reference list is available in the paper draft